**GAME OF CRAPS**

**DERIVATION FOR UNBIASED DIE**

P(1) = P(2)= P(3) = P(4) = P(5) = P(6) = 1/6

We can get a sum of 7 on the first roll of 2 die by, (1,6), (2,5),(3,4),(4,3),(5,2) , which is in simple terms, 2\*(1,6), 2\*(2,5), 2\*(3,4)

P(7 on a roll of two die) = 2\*(1/6\*1/6) + 2\*(1/6\*1/6) + 2\*(1/6\*1/6) = 6/36 =1/6

We can get a sum of 11 on the first roll of 2 die by, 2\*(5,6)

P(11 on a roll of two die) = 2\*(1/6\*1/6) = 2/36 = 1/18

P(winning on first roll of 2 die ) = 1/18 + ⅙ = 2/9

If we don’t win on the first roll of 2 dies, we keep playing till we match our score of the roll of first 2 die, in which case we win, or we roll a 7 in which case we lose.

First case is that we first roll a 4, so now we will roll again till we roll a score of 4 (in which case we will win) or roll a score of 7 in which case we will lose.

P(winning with a 4) = P(4) + P(anything other than 4 and 7 )\*P(4) + P(anything other than 4 and 7)\*P(anything other than 4 and 7)\*P(4) + ……..

=

P(anything other than 4 and 7) = P(2) + P(3) + P(5) + P(6) + P(8) + P(9) + P(10) + P(11) + P(12)

Assume P(X) is the probability of getting a score of X on the roll of die

P(2) = (1,1) = 1/36

P(3) = 2(1,2) = 2/36

P(4) = (2,2), 2(1,3) = 3/36

P(5) = 2(1,4) + 2(2,3) = 4/36

P(6) = (3,3) + 2(2,4) + 2(5,1) = 5/36

P(7) = 2\*(1,6), 2\*(2,5), 2\*(3,4) = 6/36

P(8) = (4,4) + 2(2,6) + 2(3,5) = 5/36

P(9) = 2(3,6) + 2(4,5) = 4/36

P(10) = (5,5) + 2(6,4) = 3/36

P(11) = 2(5,6) = 2/36

P(12) = (6,6) = 1/36

P(2) + P(3) + P(4) + P(5) + P(6) + + P(7) + P(8) + P(9) + P(10) + P(11) + P(12) = 36/36 = 1

Results verified

P(anything other than 4 and 7 ) = P(2) + P(3) + P(5) + P(6) + P(8) + P(9) + P(10) + P(11) + P(12) = 1 - P(4) - P(7) = 27/36

P(winning with a 4) = 3/36 + (3/36)(27/36) + (3/36)(27/36)(27/36) + …..

= (3/36) ( 1/(1-(27/36)) //geometric series formula // sum to infinity = a/(1-r ) where a if first number and r is common ratio

= ⅓

Complete probability of winning with a 4 = P( 4 on first + 4 afterwards ) = 3/36 \* ⅓ = 1/36

Repeating the same procedure for rolling 5 and winning,

P(5) = 4/36

P(anything other than 5 and 7) = 1 - P(5) - P(7) = 26/36

P(winning with 5) = 4/36(1/(1-(26/36)) = 2/5

Complete probability of winning with a 5 = 4/36 \* 2/5 = 2/45

Repeating same procedure with rolling a 6 and winning,

P(6) = 5/36

P(anything other than 6 and 7) = 1 - P(6) - P(7) = 25/36

P(winning with 6) = 5/36(1/(1-(25/36)) = 5/11

Complete probability of winning with a 6 = 5/36 \* 5/11 = 25/396

Repeating same procedure with rolling a 8 and winning,

P(8) = P(6) = 5/36

P(anything other than 8 and 7) = P(anything other than 6 and 7) = 25/36

P(winning with 8) = 5/36(1/(1 - (25/36)) ) = 5/11

Complete probability of winning with a 8 = 5/36 \* 5/11 = 25/396

Repeating same procedure with rolling a 9 and winning,

P(9) = P(5) = 4/36

P(anything other than 9 and 7) = 1 - P(5) - P(7) = 26/36

P(winning with 9) = 4/36(1/(1-(26/36)) = 2/5

Complete probability of winning with a 9 = 4/36 \* 2/5 = 2/45

Repeating same procedure with rolling a 10 and winning,

P(10) = P(4) = 3/36

P(anything other than 10 and 7) = 1 - P(10) - P(7) = 27/36

P(winning with 10) = 4/36(1/(1-(27/36)) = ⅓

Complete probability of winning with a 10 = 3/36 \* ⅓ = 1/36

Total probability of winning the game of craps = P(winning on first roll of 2 die ) + P(winning with 4) + P(winning with 5) + P(winning with 6) + P(winning with 8) + P(winning with 10) = 2/9 + 2\*(1/36) + 2\*(2/45) + 2\*(25/396) = 0.4929

**DERIVATION FOR UNBIASED DIE**

A die can be loaded with weights so that one particular outcome is more likely (e.g., rolling a 6) — and as a result, the outcome on the opposite side of the die (e.g., rolling a 1) becomes less likely. Assume that the dice are loaded such that a 6 is four times as likely as a 1, while 2, 3, 4, and 5 are each twice as likely as a 1.

For this biased die,

Assume P(1) = x , where P(1) is the probability of getting a 1 on a single roll

P(1) = 1/13

P(2)= 2/13

P(3) = 2/13

P(4) = 2/13

P(5) = 2/13

P(6) = 4/13

We can get a sum of 7 on the first roll of 2 die by, (1,6), (2,5),(3,4),(4,3),(5,2) , which is in simple terms, 2\*(1,6), 2\*(2,5), 2\*(3,4)

P(7 on a roll of two die) = 2\*(1/13\*4/13) + 2\*(2/13\*2/13) + 2\*(2/13\*2/13) = 24/169

We can get a sum of 11 on the first roll of 2 die by, 2\*(5,6)

P(11 on a roll of two die) = 2\*(2/13\*4/13) = 16/196

P(winning on first roll of 2 die ) = 24/169 + 16/169 = 40/169

If we don’t win on the first roll of 2 die, we keep playing till we match our score of the roll of first 2 die, in which case we win, or we roll a 7 in which case we lose.

First case is that we first roll a 4, so now we will roll again till we roll a score of 4 (in which case we will win) or roll a score of 7 in which case we will lose.

P(winning with a 4) = P(4) + P(anything other than 4 and 7 )\*P(4) + P(anything other than 4 and 7)\*P(anything other than 4 and 7)\*P(4) + ……..

P(winning with a 4) = 8/169 + (8/169)(137/169) + (8/169)(137/169)(137/169) + …..

= (8/169) ( 1/(1-137/169) //geometric series formula

= ¼

Complete probability of winning with a 4 = P( 4 on first + 4 afterwards ) = 8/169 \* ¼ = 2/169

Repeating the same procedure for rolling 5 and winning,

P(5) = 12/169

P(anything other than 5 and 7) = 1 - P(5) - P(7) = 133/169

P(winning with 5) = 12/169(1/(1-133/169) = ⅓

Complete probability of winning with a 5 = 12/169 \* ⅓ = 4/169

Repeating same procedure with rolling a 6 and winning,

P(6) = 16/169

P(anything other than 6 and 7) = 1 - P(6) - P(7) = 129/169

P(winning with 6) = 16/169(1/(1-129/169) = ⅖

Complete probability of winning with a 6 = 16/169 \* ⅖ = 32/845

Repeating same procedure with rolling a 8 and winning,

P(8) = 28/169

P(anything other than 8 and 7) = 1 - P(8) - P(7) = 9/13

P(winning with 8) = 28/169(1/(1-9/13) = 7/13

Complete probability of winning with a 8 = 28/169 \* 7/13 = 196/2197

Repeating same procedure with rolling a 9 and winning,

P(9) = 24/169

P(anything other than 9 and 7) = 1 - P(9) - P(7) = 121/169

P(winning with 9) = 24/169(1/(1-121/169) = ½

Complete probability of winning with a 9 = 24/169 \* ½ = 12/169

Repeating same procedure with rolling a 10 and winning,

P(10) = 20/169

P(anything other than 10 and 7) = 1 - P(10) - P(7) = 125/169

P(winning with 10) = 20/169(1/(1-125/169) = 5/11

Complete probability of winning with a 10 = 20/169 \* 5/11 = 100/1869

Total probability of winning the game of craps = P(winning on first roll of 2 die ) + P(4) + P(5) + P(6) + P(8) + P(10) = 40/169 + 2/169 + 4/169 + 32/845 + 196/2197 + 12/169 + 100/1869 = 0.524

**GAME CONSTRUCTION**

**Construction of experiment**

To create an unbiased die, I could no longer use the random number generator which produces numbers ranging from 1 to 6 since then this would not be biased. So we created a list which acts as my dice. This list contains one 1’s, two 2’s , two 3’s, two 4’s, two 5’s and four 6’s as per the specifications of the loaded dice so that P(2), P(3), P(4), P(5) = 2\* P(1) and P(6) = 4\*P(1).

I firstly have a method called fairGameSimulaton which creates the Monti Carlo data for unbiased dice game. I have a for loop running from 0 to N which ensures I carry out N replications of each experiments. The probability of each number is equal as the random number generator picks a number between 1 and 6 inclusive in every run for each of the two die, one after the other. The game is therefore played using these numbers.

I have a method called gameSimulation which creates the monti carlo data for the this loaded dice game. I have a for loop running from 0 to N which ensures that I carry out N replications of each experiment. I do a roll (which means finding the sum of two dice thrown ) twice in the program. Once it is done in the start of the program and then the sum is compared to 7 or 11, if not that then, 2, 3 or 12, and if not that as well, I start the next stage of the simulation. If the sum is 7 or 11, I set my boolean variable of winning as true and increment the number of wins. If the sum is 2,3,12, I set my boolean variable of winning as false and make no changes to the variable numberofwins. If either of these two conditions are not met, meaning my roll was 4,5,6,8,9,10, I make the roll value my POINT and I keep rolling the dice until my new roll value becomes equal to the point value. If the value equals point I increment my numberOfWins and make the boolean numberOfWins true, else I make the boolean numberOfWins false. I repeat this whole procedure N times.

**Replications: 100,000**

Estimated probability of winning with fair dice: 0.49257

Expected probability of winning with fair dice: 0.4929

Estimated probability of winning with unfair dice: 0.52409

Expected probability of winning with unfair dice: 0.524

Percentage error in fair dice = 0.030%

Percentage error in unfair dice = 0.010%